Chapter 10
Learning to Reason About Variability

Variation is the reason why people have had to develop sophisticated statistical methods to filter out any messages from the surrounding noise.
(Wild & Pfannkuch, 1999, pp. 235–236)

Snapshot of a Research-Based Activity on Variability

Students are given pairs of histograms with the assignment to discuss and determine which graph in each pair would have a higher standard deviation and why. First, students are encouraged to actually draw deviations on the histograms and draw lines from the mean to represent the number of deviations for each bar of the histogram (e.g., a bar representing five values would correspond to five lines of the same length, showing deviations from the center). Some of the histograms are easier to compare than others, such as those that have a few bars close to the center versus a histogram with most bars far from the center (see examples in Fig. 10.1). The difficult comparisons are for graphs that have the same range, the same frequencies for each bar, but different numbers of bars (representing different possible values of the variables).

Thinking about the size and number of deviations from the mean helps the students reason about which graph would have a larger standard deviation. It also helps them confront some misleading intuitions such as looking at graph A in the second example below, and initially describing this graph as having no variability, because it is confused with a time series graph or a bar graph where the height of the bars indicates that they all have the same measurement. Drawing deviations from the center helps students to realize that this histogram is different than a bar graph or time series, and that there are different deviations from the center that can be "averaged" to produce an estimate of the standard deviation.

After students compare and discuss their answers, they enter the data for each graph into *Fathom* (Key Curriculum Press, 2006) and have the standard deviations computed. They use *Fathom* to calculate the actual squared deviations from the mean for each graph. After the students have checked their answers in this way, the class discusses each pair of histograms and why the standard deviations were larger or smaller in each pair. As they do this, they construct a set of factors that appear to influence the size of the standard deviation (e.g., more bars farther from the mean) and those that do not seem to affect the size (e.g., "bumpiness" of the graph, or different heights of the bars).

J.B. Garfield, D. Ben-Zvi, *Developing Students' Statistical Reasoning*,

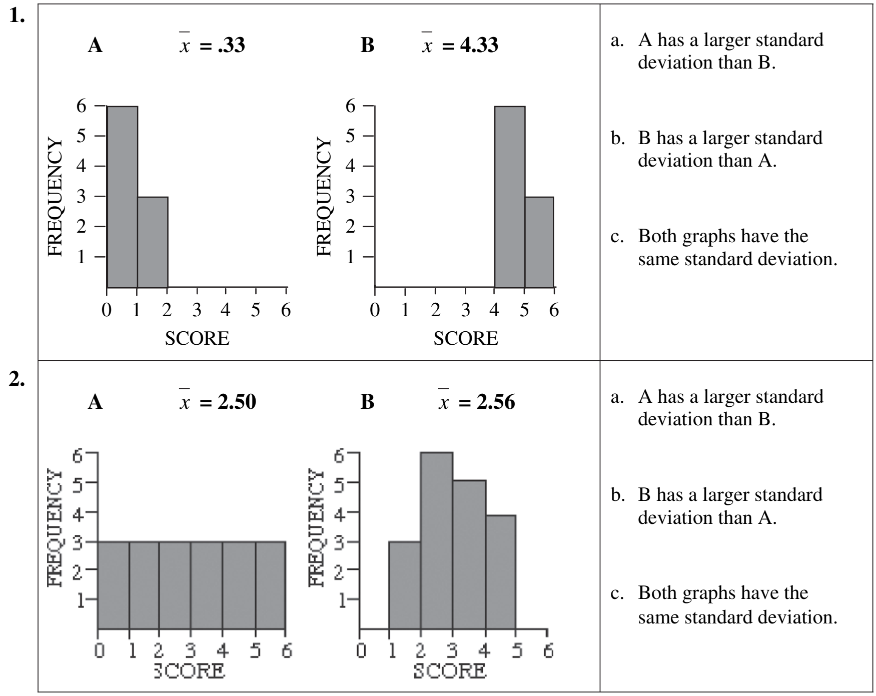

Fig. 10.1 Comparing standard deviations of pairs of histograms from the *What Makes the Standard Deviation Larger or Smaller* activity

Rationale for This Activity

The standard deviation is an important and much-used measure of variability, but one that is almost impossible for beginning statistics students to understand. While students learn what the standard deviation is and how it is calculated, they rarely have an understanding of what this measure is and how to interpret it. The activity described above is a culminating activity in the unit on variability, because it helps students recognize and integrate several sub ideas (e.g., a graphical representation of distribution, the mean of a distribution, spread, and deviation). The idea of standard deviation as an average distance from the center is developed by first having students estimate the mean of each graph, taking into account both value (on the number line) and density (frequency of each bar). Students are guided to reason about deviations from the mean and think about how values close to and far from the mean affect those deviations and squared deviations.

Entering the data from each histogram themselves (to find the actual standard deviations for each pair of graphs), helps students remember that each bar in the histogram represents one or more pieces of data of the same value, distinguishing these graphs from case value graphs (see more on this issue in Chapter 8). Seeing the squared deviations illustrates how deviations far from the mean have a greater impact on the size of the standard deviation.

The Importance of Understanding Variability

> Variability is . . . the essence of statistics as a discipline and it is not best understood by lecture. It must be experienced.
>
> (Cobb, 1992)

A major goal of most introductory statistics courses is to help students understand and be aware of the omnipresence of variability and the quantification and explanation of variability (Cobb, 1992). These two topics are also highlighted in the GAISE report (2005a, 2005b):

> *The omnipresence of variability*: Recognizing that variability is ubiquitous. It is the essence of statistics as a discipline and it is not best understood by lecture. It must be experienced.
> *The quantification and explanation of variability*: Recognizing that variability can be measured and explained, taking into consideration the following: (a) Randomness and distributions; (b) patterns and deviations (fit and residual); (c) mathematical models for patterns; (d) model-data dialogue (diagnostics).

Understanding the ideas of spread or variability of data is a key component of understanding the concept of distribution, and is essential for making statistical inferences. While students develop informal ideas of spread in the earlier unit on graphing and describing distributions, they later encounter these ideas more formally as they learn about different measures of variability (e.g., range, standard deviation, and interquartile range), what they mean, how to interpret them, how they compare to each other as statistical summaries of data, and what information they provide and do not provide, and how we use them in analyzing data.

There has been increasing attention paid to the importance of students developing an understanding of and appreciation for variability as a core component of statistical thinking (Cobb, 1992; Moore, 1998). However, it is impossible to consider variability without also considering center, as both ideas are needed to find meaning in analyzing data.

The Place of Variability in the Curriculum

The idea of spread, or variability should permeate the entire curriculum. We advocate introducing ideas of spread first informally, and later formally. Ideas of variability can be introduced the first day of class (see lessons from the data unit) and revisited in the unit on distribution, where students describe the spread or clustering of values in a graph of a distribution. When center is introduced, the idea of deviation from the mean is used to help understand the meaning of the mean, and this idea of deviation from the mean is then revisited when studying standard deviation. While range is a fairly easy concept for students to understand, standard deviation is much more difficult. Interquartile range is also a difficult concept, and best introduced in the context of comparing groups with boxplots, when it is illustrated visually by the width of the box.

It is hard to imagine a situation where one would summarize a data set using only a measure of center or using only a measure of spread. When comparing groups or making inferences, we need to look at center and spread together: the signal, and the noise around the signal. Therefore, ideas of center and spread are most often seen and used together, whether informally describing distributions, looking at theoretical models such as the normal distribution and sampling distributions, or in making inferences.

Review of the Literature Related to Reasoning About Variability

Variation vs. Variability

Before we begin to summarize current research on reasoning about variability, we want to address the question of terminology. An inconsistent use of terminology is noticeable in research studies about variability. While some use "variability" and "variation" interchangeably, others distinguish between the meanings of these two words. Reading and Shaughnessy (2004) suggest the following distinctions: variation is a noun describing the act of varying, while variability is a noun form of the adjective "variable," meaning that something is likely to vary or change. Since this distinction has not yet been agreed upon in the statistics education research community, we note this argument but have chosen to use the term *variability* as the general, omnibus term for these ideas in this chapter.

The Emergent Research About Variability

Recent discussions in the statistics education community have drawn attention to the fact that statistics text books, instruction, public discourse, as well as research have been overemphasizing measures of center at the expense of variability (e.g., Shaughnessy, 1997). Instead, there is a growing consensus to emphasize general distributional features such as shape, center, and spread and the connections among them in students' early experiences with data. It is also suggested to focus students' attention on the nature and sources of variability of data in different contexts, such as variability in a particular data set, outcomes of random experiments, and sampling (Shaughnessy, Watson, Moritz, & Reading, 1999; Gould, 2004). These views are supported by a review of several studies by Konold and Pollatsek (2002) that has shown that "the notion of an average understood as a central tendency is inseparable from the notion of spread" (p. 263). Their well-known metaphor for data as *signal and noise* implies that students should come to see statistics as "the study of noisy processes – processes that have a signature, or signal" (p. 260).

Difficulties in Understanding Variability

Despite the widespread belief in the importance of this concept, current research demonstrates that it is extremely difficult for students to reason about variability and that we are just beginning to learn how reasoning about variability develops (Garfield & Ben-Zvi, 2005). Understanding variability has both informal and formal aspects, moving from understanding that data vary (e.g., differences in data values) to understanding and interpreting formal measures of variability (e.g., range, interquartile range, and standard deviation). While students can learn how to compute formal measures of variability, they rarely understand what these summary statistics represent, either numerically or graphically, and do not understand their importance and connection to other statistical concepts. What makes the understanding of the concept even more complex is that variability may sometimes be desired and of interest, and sometimes be considered error or noise (Gould, 2004; Konold & Pollatsek, 2002), as well as the interconnectedness of variability to concepts of distribution, center, sampling, and inference (Cobb et al., 2003b).

These difficulties are evident, for example, in a series of interview studies with undergraduate students who had earned a grade of A in their college statistics course, Mathew and Clark (2003) found that students could not remember much at all about the standard deviation. In another interview study of introductory statistics students' conceptual understanding of the standard deviation, delMas and Liu (2005) designed a computer environment to promote students' ability to coordinate characteristics of variation of values about the mean with the size of the standard deviation as a measure of that variation. delMas and Liu found that students moved from simple, one-dimensional understandings of the standard deviation that did not consider variation about the mean to more mean-centered conceptualizations that coordinated the effects of frequency (density) and deviation from the mean.

In a study investigating students' statistical reasoning, using the Statistical Reasoning Assessment (SRA), Garfield (2003) found that even students in introductory classes that were using reform textbooks, good activities, and high-quality technology had significant difficulty reasoning about different aspects of variability, such as representing variability in graphs, comparing groups, and comparing the degree of variability across groups.

Developing Students Reasoning About Variability

A variety of contexts have been used in statistics education to study students' reasoning about variability at all age levels. For example, in a study of elementary students, Lehrer and Schauble (2007) contrast students' reasoning about variability in two contrasting contexts: (a) measurement and (b) "natural" (biological). When fourth-graders were engaged in measuring the heights of a variety of objects, distribution emerged as a coordination of their activity. They were able to invent statistics as indicators of stability (e.g., center corresponded to "real" length) and variation of measure (e.g., spread corresponded to sources of error such as tool, person, trial-to-trial). In the context of natural variation activity (growth of plants), these same students (now fifth-graders) had difficulties handling sources of natural variation and related statistics. Activities that promoted investigations of sampling (e.g., "what would be likely to happen to the distribution of plant heights if we grew them again") and comparing distributions (e.g., "how one might know whether two different distributions of height measurements could be considered 'really' different") were found useful in developing students' understanding of variability.

In a design research conducted with students in grades 7 and 8 (Bakker, 2004b), instructional activities that could support coherent reasoning about key concepts such as variability, sampling, data, and distribution were developed. Two instructional activities were found to enable a conceptual growth: A "growing a sample" activity that had students think about what happens to the graph when bigger samples are taken, and an activity requiring reasoning about shape of data.

The advantage in discussing ideas of variability in connection with ideas of center was described by Garfield et al. (2007). In this study with undergraduate students, results indicated that students could develop ideas of "a lot" or "a little" variability when asked to make and test conjectures about a series of variables measuring minutes per day spent on various activities (e.g., time spent studying, talking on the phone, eating, etc.). They also found that by having students reason abut the distributions of these variables informally, they could move them toward comparisons of formal measures of variability (e.g., standard deviation, range, and interquartile range).

Other contexts examined include variability in data (Ben-Zvi, 2004a; Groth, 2005; Konold & Pollatsek, 2002; Petrosino, Lehrer, & Schauble, 2003), bivariate relationships (Cobb et al., 2003b; Hammerman & Rubin, 2003), comparing groups (Ben-Zvi, 2004b; Biehler, 2001; Lehrer & Schauble, 2002; Makar & Confrey, 2005), probability contexts (Reading & Shaughnessy, 2004), measures of spread such as the standard deviation (delMas & Liu, 2005), and sampling (Chance et al., 2004; Watson, 2004). These studies are mostly exploratory and qualitative, and their research goal is often to explore what and how students come to understand ideas of variability in the different contexts. The kinds of questions and activities used in these studies suggest ways we can help students develop reasoning about variability across an entire course as well as assess informal and formal aspects of students' understanding of variability.

Levels of Reasoning About Variability

Based on results from a large sample of students on a survey of variability tasks, Watson et al. (2003) propose a model of students' reasoning about variability. Using the SOLO model (Biggs & Collis, 1982), they qualitatively describe four hierarchical levels of progressively sophisticated understanding of reasoning about variability: Prestructural, Unistructural, Multistructural, and Relational. They suggested that this scale might be useful in tracking student improvement over time and in relation to particular sequences of learning activities. The descriptions of these levels, as well as the work on informal and formal ideas of variability (Garfield et al., 2007) suggest the need for carefully designed activities to lead students to develop higher or more formal levels of reasoning.

Implications of the Research: Teaching Students to Reason About Variability

Noticeably lacking in the current research literature are studies of how to best impact the learning of college students who are typically introduced to variability in a unit of descriptive statistics, following the units of graphing univariate data and measures of center. Measures of variability (or spread) are then introduced, and students learn to calculate and briefly interpret them. Typically, only the formal notion of variability as measured by three different statistics (i.e., the range, interquartile range, and standard deviation) is taught. Students often do not hear the term "variability" stressed again until a unit on sampling, where they are to recognize that the variability of sample means decreases as sample size increases. When students are introduced to statistical inference, variability is then treated as a nuisance parameter because estimating the mean becomes the problem of importance (Gould, 2004).

Given this typical introduction in textbooks and class discussion, it is not surprising that few students actually develop an understanding of this important concept. Good activities and software tools designed to promote an understanding of variability do exist. However, they are typically added to a lesson or given as an assignment instead of being integrated into a plan of teaching, and their impact on student understanding has not been subjected to systematic study. So, while there have been positive changes in introductory statistics classes, they still fall short of giving students the experiences they need to develop statistical thinking and a deep understanding of key statistical concepts.

We would like our students to follow the way statisticians think about variability. When statisticians look at one or more data sets, they often appraise and compare the variability informally and then formally, looking at appropriate graphs and descriptive measures. They look at both the center of a distribution as well as the spread from the center, often referring to more than one representation of the data to lead to better interpretations. Statisticians are also likely to consider sources of variability, including the statistical and measurement processes by which the data were collected.

Konold and Pollatsek (2002) offer the following suggestions about how we might help students and future teachers develop ideas of the signal-noise perspective of various statistical measures:

1. Using processes involving repeated measures;
2. Explorations of stability such as drawing multiple samples from a known population and evaluating particular features, such as the mean, across these replications;
3. Comparing the relative accuracy of different measurement methods;
4. Growing samples – students observe a distribution as the sample gets larger;
5. Simulating processes – students investigate why many noisy processes tend to produce mound-shaped distributions;
6. Comparing groups; or
7. Conducting experiments.

Garfield and Ben-Zvi (2005) outline a list of increasingly sophisticated ideas for constructing "deep understanding" of variability. This list offers a sequence through which students may be guided to develop a deep understanding of this concept, as shown below:

1. Developing intuitive ideas of variability
2. Describing and representing variability with numerical measures
3. Using variability to make comparisons
4. Recognizing variability in special types of distributions
5. Identifying patterns of variability in fitting models
6. Using variability to predict random samples or outcomes
7. Considering variability as part of statistical thinking

Progression of Ideas: Connecting Research to Teaching

Introduction to the Sequence of Activities to Develop Reasoning About Variability

Table 10.1 shows a series of steps that can be used to help students first build informal and then formal ideas of variability. These ideas are first introduced in earlier units on data, distribution, and center. Then the formal idea of standard deviation is introduced and is used to examine and reason about data. The concept of interquartile range is introduced in the later unit on comparing groups, a unit that helps connect ideas of center and spread visually and for the purpose of comparing sets of data to answer a research question. The idea of variability is visited again in the unit on models, when the normal distribution is introduced and the unique characteristics of the mean and standard deviation are shown as part of the Empirical Rule. The interconnections of center and spread are also demonstrated in the sampling, statistical inference, and covariation units. Each time the basic idea of variability is explicitly revisited in that particular context, emphasized, and discussed.

Introduction to the Lessons

While students have been informally introduced to the idea of spread and range earlier, this set of lessons looks more closely at variability and the standard deviation. First, students collect measurements to help them recognize two ways of looking at variability, as noise and as diversity. They informally think about a measure of spread from the center. The second lesson helps develop the idea of standard deviation and encourages students to reason about how this statistics is used to measure and represent variability and factors that affect the standard deviation, making it larger or smaller.

Table 10.1 Sequence of activities to develop reasoning about variability[1]

Milestones: ideas and concepts	Suggested activities
Informal ideas prior to formal study of variability	
• Data vary. Values of a variable illustrate variability	• Meet and Greet Activity (Lesson 1, Data Unit, Chapter 6)
• Variability in Results from a random experiment	• Activities in Lessons 1 and 2, Statistical Models Unit (Chapter 7)
• Informal idea of spread of data by examining a graph or comparing graphs	• Distinguishing Distributions Activity (Lesson 1, Distributions Unit, Chapter 8)
• Range as a simple measure of spread	❖ An activity where students describe distribution, and note range as a measure of spread. (The symbol ❖ indicates that this activity is not included in these lessons.)
Formal ideas of variability	
• Two ideas of variability: diversity or measurement error	• How Big is Your Head Activity (Lesson 1: "Variation")
• Sources of variability, a lot and a little variability	• How Big is Your Head Activity (Lesson 1)
• Averaging deviations from the mean as a measure of spread	• Comparing Hand Spans Activity (Lesson 2: "Reasoning about the Standard Deviation")
• Standard deviation as a measure of average distance from the mean	• Comparing Hand Spans Activity (Lesson 2)
• Understanding factors that cause the standard deviation to be larger or smaller	• What Makes the Standard Deviation Larger or Smaller Activity (Lesson 2)
• How center and spread are represented in graphs?	❖ An activity where students match a set of graphs to the corresponding set of statistics
Building on formal ideas of variability in subsequent topics	
• Range and IQR in a boxplot	• How Many Raisins in a Box Activity (Lesson 1, Comparing Groups Unit, Chapter 11)
• Variability within a group and variability between groups	• Gummy Bears Activity (Lesson 2, Comparing Groups Unit, Chapter 11)
• What makes the range and IQR larger and smaller?	• How do Students Spend their Time Activity (Lesson 4, Comparing Groups Unit, Chapter 11)
• Understanding how and why center and spread are used to compare groups	• How do Students Spend their Time Activity (Lesson 4, Comparing Groups Unit, Chapter 11)
• Role of mean and standard deviation in describing location of values in a normal distribution	• Activities in Lesson 3, Statistical Models Unit (Chapter 7)
• Understanding why and how variability decreases as sample size increases in sampling distributions	• The Central Limit Theorem Activity (Lesson 3, Samples and Sampling Unit, Chapter 12)
• Understanding ideas of variability between and within groups when comparing samples of data	• Gummy Bears Revisited Activity (Lesson 4, Statistical Inference Unit, Chapter 13)
• Variability of data in a bivariate plot	• Interpreting Scatterplots Activity (lesson 1, Covariation Unit, Chapter 14)

[1] See page 391 for credit and reference to authors of activities on which these activities are based.

Lesson 1: Variation

This lesson is designed to help students reason informally about variability. Students compare measurements for two sets of repeated measurements, to discover two kind of variability: (1) variability as an error of measurement (repeated measures of the same head circumference); and (2) variability as an indicator of diversity (measurements of different people's head circumferences). Students are then introduced to the concept of signal and noise, and discuss the stability of the mean as more data are collected. Student learning goals for the lesson include:

1. Understand different types (sources) of variability (when it's desired and when it's noise).
2. Understand the ideas of mean as signal and variability as noise, from repeated measurements in an experiment.
3. Understand that it is desirable to reduce variability in measurement (by using experimental protocols).

Description of the Lesson

Students are asked to think about variability in the class, and in particular, of head sizes. In the *How Big is Your Head* activity, they plan a method to measure the circumference of each of their heads, keeping track of the decisions they make about measuring. A class discussion about this results in a common protocol to use. Students are given a measuring tape to use, and they measure each of their heads using the protocol established and they record the data on the board (or on a computer spreadsheet).

Next, as a class, the students choose one person who will have their head measured by every student in the class. These measurements are also recorded for the class. Students then work with a partner to obtain a set of additional body measurements (all in centimeters) listed in Fig. 10.2. These data will be used in other activities in the course. These data are later entered in *Fathom*.

Body Data Collection

Height (with shoes on): __________

Arm Span (from fingertip to fingertip with arms out-stretched): __________

Kneeling Height: __________

Hand Length (from the wrist to the tip of the middle finger): __________

Hand Span (from the tip of the thumb to the tip of the pinkie while hand is stretched): __________

Fig. 10.2 Record sheet for the body measurements survey

Fathom is used to create a plot of students' head sizes so that they may be examined and summarized. Unusual values are examined and discussed to see if they are legitimate or the result of a faulty measurement process.

Students then select two numbers that seem reasonable for completing the following sentence. (Note: There is more than one reasonable set of choices.)

> *The typical head circumference for students in this class is about ________ cm give or take about ________ cm.*

These answers as discussed and lead to a discussion of possible reasons for the variability in the measurements of students' head circumferences. Students then consider whether the observed variability could be reduced and if so, how that might happen. They offer suggestions for ways to make the measurements more standard.

Next, the class examines data for the repeated measurements of one student's head circumference. These data are entered into *Fathom* and are graphed and summarized in terms of shape, center, and spread. This graph is then compared to the first graph of all students' head circumferences and reasons for the differences are discussed. This time, the students suggest that the variability is solely due to the measurement process and talk about ways to reduce that variability.

A class discussion of the difference between these two sets of measurements of head circumference includes the different types and sources of variability, and when we might expect (and accept) variability in measurements and when we want to keep it as small as possible. The concepts of *signal and noise* are revisited, and the idea of variability as noise in the case of the repeated measurements of one head is discussed.

A wrap-up discussion includes suggestions for different sources of variability in data; the two kinds of variability are: "diversity" (spice of life) and "error or noise". Students are asked which type of variability we would like to have large and which we would like to have small, and why. They finally come up with some other examples of signal and noise, and to consider what is important in examining and interpreting signal and noise when we explore data.

Lesson 2: Reasoning About the Standard Deviation

This lesson encourages students to reason about the standard deviation. Students begin by visualizing and estimating average distances in an activity involving hand spans. The next part of the lesson is designed to help students improve their reasoning about and understanding of variability by thinking about what a standard deviation is and applying that knowledge to determine which of two graphs has a higher standard deviation. Student learning goals for the lesson include:

1. Understand and informally estimate deviations from the mean and "typical" deviation from the mean.
2. Understand standard deviation as a measure of spread.

3. Understand what makes standard deviations larger or smaller, what types of graphs reveal different amounts of variation.
4. Reason about connections between measure of center and spread, and how they are revealed in graphical representations of data.

Description of the Lesson

In the first activity, *Comparing Hand Spans*, students examine and compare their hands and think about variability in hand spans. Students find the hand span for every person in their group. They use a dot plot to examine how these values vary. They suggest two sources of variability in these measurements, i.e., two reasons why the measurements are not all the same.

Next, students record initials above the dots to identify each case. They find the mean and mark it with a wedge (▲) below the correct place on the number line. They estimate how far each of their hand span measurements is from the mean of their group. They make a second dot plot, this time of the differences (deviations) from the mean for each student in their group, and find the mean of these differences (deviations). Using the idea of deviations from the mean, students suggest a "typical" distance (deviation) from the mean.

Fathom is used to re-create the dot plot and to check their calculations and to compute the standard deviation of the group's hand span data. Students compare the actual standard deviation to the "typical" distance (deviation) the group found earlier and to speculate about the difference in these values.

Students then access the entire set of Hand Spans for the class that were gathered in the pervious activity and find the standard deviation of these measurements. This statistic is compared to the standard deviation of hand spans for the small group of four originally produced, and differences are discussed. Finally, students discuss the idea of a "typical" deviation and the standard deviation.

The second activity *What Makes the Standard Deviation Larger or Smaller* continues the discussion of a typical, or standard, deviation from the mean. First, students examine the following dot plot (Fig. 10.3), which has the mean marked by a vertical line. Students consider how large the deviations would be for each data point (dot).

Next students draw in the plot each deviation from the mean as shown below (Fig. 10.4).

Next, students reason about the average size (length) of all of those deviations, and use this to estimate the standard deviation. They draw the estimated length of the standard deviation. This process is repeated with a second dot plot as shown below (Fig. 10.5).

Then students are given a histogram (Fig. 10.6) and they use the same process, thinking about dots "hidden" by the bars, to draw and estimate the length of the standard deviation. The mean of the data set is given (2.57). Students are encouraged to draw in the appropriate number of dots in each bar of the histogram to make sure they have the appropriate number of deviations.

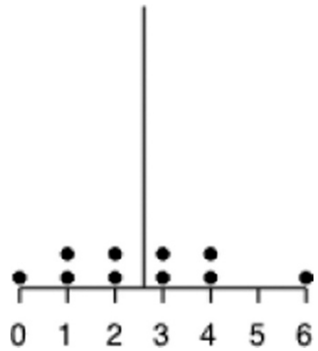

Fig. 10.3 Dot plot from the *What Makes the Standard Deviation Larger or Smaller* activity

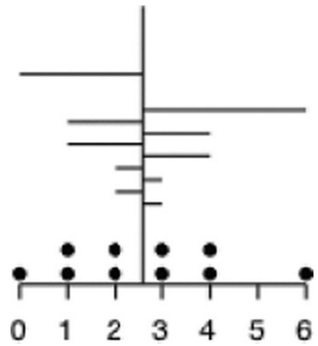

Fig. 10.4 Drawing deviations from the mean in a dot plot from the *What Makes the Standard Deviation Larger or Smaller* activity

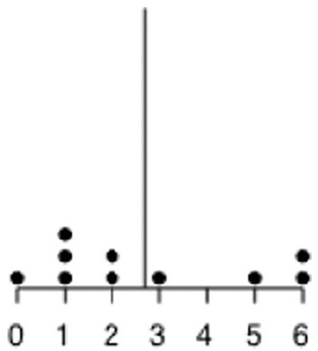

Fig. 10.5 Second dot plot from the *What Makes the Standard Deviation Larger or Smaller* activity

Students are then given six pairs of histograms, for which they are to try to determine which graph in the pair would have a larger standard deviation or would they be the same, and why. The mean for each graph is given just above each histogram. In doing so, students try to identify the characteristics of the graphs that make the standard deviation larger or smaller. Two such pairs of histograms are shown above in Fig. 10.1.

After students complete the set of comparisons, their answers can be discussed and compared as a class and correct answers provided (e.g., the actual size of

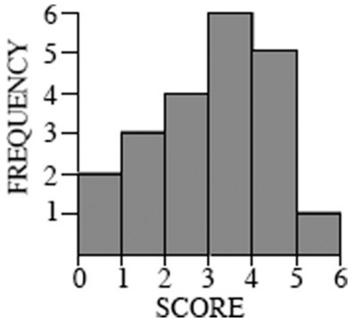

Fig. 10.6 A histogram from the *What Makes the Standard Deviation Larger or Smaller* activity

the standard deviation for each graph in the activity). Students elaborate on which graphs were harder to compare, which were easier and why.

In a wrap-up discussion, students comment on why we need measures of variability in addition to measures of center, and why variability is so important in data analysis. They speculate on why variability is the basis of statistical analysis and how we represent and summarize variability.

Summary

The two lessons in this unit focus mainly on the ideas of types of variability and the meaning of the standard deviation. If students can develop an understanding of this important measure of spread, it will help them learn and reason about the related concept of sampling error in the unit on sampling (Chapter 12) and margin of error in the unit on inference (Chapter 13). The next chapter (Chapter 11) introduces the range and interquartile range as measures of spread in the context of using boxplots to compare groups.